THERMO FIELD DYNAMICS FOR QUANTUM FIELDS WITH CONTINUOUS MASS SPECTRUM APPLIED TO NUCLEAR PHYSICS*†

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ABSTRACT

Transport coefficients are obtained by incorporating a gauge principle into thermo field dynamics of inhomogeneous systems. In contrast to previous derivations, neither imaginary time arguments nor perturbation theory in powers of a coupling constant are used in the calculation. Numerical values are calculated for the pion component in hot nuclear matter.

1. Introduction

In the presence of homogeneous matter, or in a heat bath, the irreducible representations of the space-time symmetry group are characterized by a continuous energy parameter rather than by a mass-shell constraint. It follows, that physical "particles" are unstable and therefore a naive perturbation theory starting from free on-shell particles may lead to inconsistencies. To circumvent this problem, a perturbative expansion in terms of generalized free fields may be defined, cf. ref. [1] and papers quoted there.

Thermo field dynamics (TFD, see ref. [2]) is a necessary input to this solution, since in contrast to the Schwinger-Keldysh (closed-time-path, CTP) method [3] of quantum statistical mechanics, it possesses two different (anti-)commuting representations of the underlying field algebra. Apart from this mathematical aspect however, TFD also contains a concept which extrexemly simplifies its practical application: Physical excitations are obtained as superposition of *thermal quasiparticle* states. Due to the facts outlined above, this must be a continuous superposition, with a weight function that is nothing but the *spectral function* of a field theory.

To establish the notation, we first discuss TFD for a single bosonic oscillator, introducing canonical creation and annihilation operators $a^\dagger$, a, $\widetilde{a}^\dagger$, $\widetilde{a}$ for the two representations. The a, $\widetilde{a}$ operators annihilate the physical vacuum, and the two sets are transformed into another by means of an anti-unitary mapping, called the

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tilde conjugation [2]. One may construct these representations in the Liouville space of the oscillator, where the generator of time evolution is

$$\widehat{H} = \omega\, \widehat{N}, \qquad \widehat{N} = a^\dagger a - \widetilde{a}^\dagger \widetilde{a}\ . \tag{1}$$

This operator possesses a continuous symmetry, since it is invariant under Bogoliubov transformations to a new set of operators

$$\begin{pmatrix} \xi \\ \widetilde{\xi}^\sharp \end{pmatrix} = \mathcal{B} \begin{pmatrix} a \\ \widetilde{a}^\dagger \end{pmatrix} \quad \begin{pmatrix} \xi^\sharp \\ -\widetilde{\xi} \end{pmatrix}^T = \begin{pmatrix} a^\dagger \\ -\widetilde{a} \end{pmatrix}^T \mathcal{B}^{-1}\ , \tag{2}$$

where $\mathcal{B}$ is a (real) 2×2 matrix with determinant 1. These matrices form the symplectic group in two dimensions Sp(2) = SU(1,1), it is a noncompact and nonabelian group [4].

In thermo field dynamics, the new set of operators obeys thermal state conditions:

$$\xi \| W^R \rangle\!\rangle = 0\ , \quad \widetilde{\xi} \| W^R \rangle\!\rangle = 0\ , \quad \langle\!\langle W^L \| \xi^\sharp = 0\ , \quad \langle\!\langle W^L \| \widetilde{\xi}^\sharp = 0\ , \tag{3}$$

where $\langle\!\langle W^L\|$ and $\|W^R\rangle\!\rangle$ are left and right thermal state. It can be shown, that a particularly simple parameterization of TFD is obtained with

$$\|W^R\rangle\!\rangle = \|W\rangle\rangle\ , \qquad \langle\!\langle W^L\| = \langle\langle 1\|\ , \quad \mathcal{B} = \begin{pmatrix} 1+n & -n \\ -1 & 1 \end{pmatrix}\ . \tag{4}$$

Here, $\|1\rangle\rangle$ is the Liouville vector associated with the unit operator, and $\|W\rangle\rangle$ is the Liouville vector associated with the density matrix W of the quantum system. Statistical averages for an operator $\mathcal{E}$ are calculated as $\langle\langle 1\|\mathcal{E}\otimes 1\|W\rangle\rangle$, i.e., as a matrix element instead of a trace over the Hilbert space of the oscillator.

2. Gauging the Bogoliubov symmetry

The Bogoliubov symmetry discussed above is a *global* symmetry, which for the simple case of a single oscillator means that it is time-independent. We now consider the question of a *local* Bogoliubov symmetry, which may arise in case the transformation matrix depends on some external parameters [5]. For convenience, we consider these external parameters as a vector $\boldsymbol{r}$ in some abstract space.

Requiring the locality of a former global symmetry is called, in accordance with the methods applied to fundamental forces of nature, the *gauging* of the Bogoliubov symmetry. This name however is more than a simple analogy: It is known, that even in simple dynamical systems structures arise, that most closely resemble the coupling of the system to an external gauge potential [6].

The physical particle operators depend on the parameter vector $\boldsymbol{r}$. Hence, the former will change with time, for infinitesimal changes of $\boldsymbol{r}$ according to

$$\boldsymbol{r} \longrightarrow \boldsymbol{r} + \delta\boldsymbol{r}: \quad \begin{pmatrix} a(t) \\ \widetilde{a}^\dagger(t) \end{pmatrix} \longrightarrow (1 + \mathcal{P}(\boldsymbol{r})\delta\boldsymbol{r}) \begin{pmatrix} a(t) \\ \widetilde{a}^\dagger(t) \end{pmatrix}$$

$$\begin{pmatrix} a^\dagger(t) \\ -\widetilde{a}(t) \end{pmatrix}^T \longrightarrow \begin{pmatrix} a^\dagger(t) \\ -\widetilde{a}(t) \end{pmatrix}^T (1 - \mathcal{P}(\boldsymbol{r})\delta\boldsymbol{r}) \ . \tag{5}$$

$\mathcal{P}$ is a vector in $\boldsymbol{r}$-space, $\mathcal{P} = (\mathcal{P}^1, \mathcal{P}^2, \ldots)$, each component forming a 2×2-matrix.

This implies, that the change of the physical particles with the external parameters has a generator

$$\widehat{\mathcal{Q}} = \mathrm{i} \begin{pmatrix} a^\dagger(t) \\ -\sigma\widetilde{a}(t) \end{pmatrix}^{\mathrm{T}} \mathcal{P}(\boldsymbol{r})\, \dot{\boldsymbol{r}} \begin{pmatrix} a(t) \\ \widetilde{a}^\dagger(t) \end{pmatrix} \ . \tag{6}$$

The total time evolution of the system is generated by the sum of $\widehat{H}$ and $\widehat{\mathcal{Q}}$, i.e., the Liouville equation for an operator $\mathcal{E}$ reads

$$\mathrm{i}\frac{\partial}{\partial \mathrm{t}} \left\| \mathcal{E}(\mathrm{t}) \right\rangle\!\!\rangle = \left(\widehat{\mathrm{H}} + \widehat{\mathcal{Q}}\right) \left\| \mathcal{E}(\mathrm{t}) \right\rangle\!\!\rangle \ . \tag{7}$$

Now consider the thermal quasi-particle operators, which under an $\boldsymbol{r}$-evolution change with a *different* $\widehat{\mathcal{Q}}$. From eqns. (2) and (5) follows, that it is obtained as

$$\mathcal{P}'(\boldsymbol{r}) = \mathcal{B}(\boldsymbol{r})\, \mathcal{P}(\boldsymbol{r})\, \mathcal{B}^{-1}(\boldsymbol{r}) + \mathcal{B}(\boldsymbol{r}) \left(\nabla_{\boldsymbol{r}} \mathcal{B}^{-1}(\boldsymbol{r})\right) \ . \tag{8}$$

Hence, under a change of the operator description mediated by a Bogoliubov transformation, $\mathcal{P}$ transforms inhomogeneously like a gauge potential. The substitution of the ξ-operators for the a-operators is therefore, in this new language, a *gauge transformation*. Finally, we require diagonality of the hamiltonian for thermal quasi-particles, i.e.,

$$\left(\widehat{H} + \widehat{\mathcal{Q}}\right)[\xi] = \omega \left(\xi^\ddagger \xi - \widetilde{\xi}^\ddagger \widetilde{\xi}\right) \ . \tag{9}$$

Returning to eqn. (8), this means that $\mathcal{P}'$ has to vanish. This requirement fixes the connection between the physical particles and the statistical quasi-particles unambiguously: The change of the physical particle operators with the external parameters according to (5) has a gauge potential given by

$$\mathcal{P}(\boldsymbol{r}) = -\left(\nabla_r\, \mathcal{B}^{-1}(\boldsymbol{r})\right) \mathcal{B}(\boldsymbol{r}) \ . \tag{10}$$

To summarize these considerations: *Gauging* the Bogoliubov symmetry of the time evolution in Liouville space, while keeping the thermal quasi-particle operators fixed, introduces a coupling of the physical particle representation to gradients in the external parameters.

3. Interacting fields with local Bogoliubov symmetry

An interacting charged scalar field at a fixed time can be expanded into momentum eigenmodes as

$$\phi_x = \int \frac{d^3\boldsymbol{k}}{\sqrt{(2\pi)^3}} \left(a^\dagger_{k-}(t)\ \mathrm{e}^{-\mathrm{i}\boldsymbol{k}\boldsymbol{x}} + a_{k+}(t)\ \mathrm{e}^{\mathrm{i}\boldsymbol{k}\boldsymbol{x}}\right) \ , \tag{11}$$

and similarly for the tilde-field. $\boldsymbol{k}$ is the three-momentum of the modes, and in this notation $a^\dagger_{k-}(t)$ creates a negatively charged excitation with momentum $\boldsymbol{k}$, while $a_{k+}(t)$ annihilates a positive charge. We will henceforth distinguish the two different charges by an additional index $l = \pm$ whenever possible.

In general, the commutation relations of the a-operators are not known: They can be very complicated due to the interaction. However, for the purpose of calculating bilinear expectation values, e.g. propagators, it is sufficient to know the expectation value of the commutator functions. Hence, for this purpose, the scalar field operators may be considered as generalized free fields, with an expansion into modes with definite energy and momentum that are created and annihilated by a new set of operators. The details of the expansion procedure in general are outlined in refs. [1,7,8], its generalization to inhomogeneous states in [10,11]. Here it is sufficient to write down the generalization of eqn. (2)

$$\begin{pmatrix} a_{kl}(t) \\ \widetilde{a}^\dagger_{kl}(t) \end{pmatrix} = \int_0^\infty dE \int d^3\boldsymbol{q}\ \left(\mathcal{A}_l(E,(\boldsymbol{q}+\boldsymbol{k})/2)\right)^{1/2} \left(\widetilde{\mathcal{B}}^{-1}_l(E,\boldsymbol{q},\boldsymbol{k})\right)^\star \begin{pmatrix} \xi_{Eql} \\ \widetilde{\xi}^\sharp_{Eql} \end{pmatrix} \mathrm{e}^{-\mathrm{iEt}} \ . \quad (12)$$

$\mathcal{A}_l(E,\boldsymbol{k})$ is a positive weight function with support on the positive energy axis, and the transformation matrix has the form

$$\widetilde{\mathcal{B}}_l(E,\boldsymbol{q},\boldsymbol{k}) = \begin{pmatrix} (\delta^3(\boldsymbol{q}-\boldsymbol{k}) + N_l(E,\boldsymbol{q},\boldsymbol{k})) & -N_l(E,\boldsymbol{q},\boldsymbol{k}) \\ -\delta^3(\boldsymbol{q}-\boldsymbol{k}) & \delta^3(\boldsymbol{q}-\boldsymbol{k}) \end{pmatrix} \quad (13)$$

similar to the matrix in eqn. (4). The Bogoliubov parameter N_l appearing here is the Fourier transform of a space-local quantity

$$N_l(E,\boldsymbol{q},\boldsymbol{k}) = \frac{1}{(2\pi)^3} \int d^3\boldsymbol{z}\ \mathrm{e}^{-\mathrm{i}(\boldsymbol{q}-\boldsymbol{k})\boldsymbol{z}}\ n_l(E,(\boldsymbol{q}+\boldsymbol{k})/2,\boldsymbol{z}) \ . \quad (14)$$

The thermal state conditions are similar to eqn. (4), but hold for both charges and for each value of $E,\boldsymbol{k}$. The time derivative of the a-operators in the Heisenberg picture then is

$$\begin{aligned} \mathrm{i}\frac{\partial}{\partial \mathrm{t}} \begin{pmatrix} a_{kl}(t) \\ \widetilde{a}^\dagger_{kl}(t) \end{pmatrix} &= \Omega_{kl} \begin{pmatrix} a_{kl}(t) \\ \widetilde{a}^\dagger_{kl}(t) \end{pmatrix} \\ &+ \int_0^\infty dE \int d^3\boldsymbol{q}\ \frac{\left(\mathcal{A}_l(E,(\boldsymbol{k}+\boldsymbol{q})/2)\right)^{1/2}}{Z_{ql}}\ E\, N_l(E,\boldsymbol{k},\boldsymbol{q})\ \times \\ &\quad \left(\left(\mathcal{A}_l(E,\boldsymbol{k})\right)^{1/2} - \left(\mathcal{A}_l(E,\boldsymbol{q})\right)^{1/2}\right) \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} a_{ql}(t) \\ \widetilde{a}^\dagger_{ql}(t) \end{pmatrix} \ , \quad (15) \end{aligned}$$

which we use to determine the operator $\widehat{\mathcal{Q}}$. The Z-factor is defined as

$$Z_{kl} = \int_0^\infty dE\, \mathcal{A}_l(E, \boldsymbol{k}) \;, \tag{16}$$

and $\Omega_{kl} = 1/(2Z_{kl})$. Obviously the additional term in the time evolution vanishes for translationally invariant n_l, i.e., when $N_l(E, \boldsymbol{k}, \boldsymbol{q})$ is proportional to $\delta^3(\boldsymbol{k} - \boldsymbol{q})$. This is consistent with eqn. (10), henceforth the generator $\widehat{\mathcal{Q}}$ is

$$\widehat{\mathcal{Q}} = \sum_{l=\pm} \int d^3\boldsymbol{k}\, d^3\boldsymbol{q} \begin{pmatrix} a^\dagger_{kl}(t) \\ -\widetilde{a}_{kl}(t) \end{pmatrix}^T \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \mathcal{P}_l(\boldsymbol{k}, \boldsymbol{q}) \begin{pmatrix} a_{ql}(t) \\ \widetilde{a}^\dagger_{ql}(t) \end{pmatrix} \;. \tag{17}$$

It has a kernel

$$\begin{aligned} \mathcal{P}_l(\boldsymbol{k}, \boldsymbol{q}) \;\; = \;\; & \int_0^\infty dE \left\{ \frac{(\mathcal{A}_l(E, (\boldsymbol{k}+\boldsymbol{k})/2))^{1/2}}{Z_{kl}\, Z_{ql}} \right\} E\, N_l(E, \boldsymbol{k}, \boldsymbol{k}) \\ & \left((\mathcal{A}_l(E, \boldsymbol{k}))^{1/2} - (\mathcal{A}_l(E, \boldsymbol{k}))^{1/2} \right) \;. \end{aligned} \tag{18}$$

The dimension of the numerical factors in this kernel is mass^2, due to the energy normalization factors contained in the a-operators for the interacting field (11).

4. Transport coefficients

Observables are expressed as functionals of the interacting fields – and therefore as functionals of the operators a, $a^\dagger$. Hence, although the state we consider is stationary in terms of the basis defined by the ξ-operators, momentum mixing as introduced above will result in a non-trivial time evolution of *physical* quantities. Two quantities we are interested in are the particle current operator of the complex scalar field and its expectation value,

$$\widehat{\boldsymbol{j}}(t, \boldsymbol{x}) = \mathrm{i}\, (\phi^\star_{\mathrm{x}} \nabla \phi_{\mathrm{x}} - \phi_{\mathrm{x}} \nabla \phi^\star_{\mathrm{x}}) \;, \quad \boldsymbol{j}(\mathrm{t}, \boldsymbol{x}) = \delta \left\langle \widehat{\boldsymbol{j}}(\mathrm{t}, \boldsymbol{x}) \right\rangle \tag{19}$$

and the energy current

$$\widehat{\boldsymbol{E}}(t, \boldsymbol{x}) = (\partial_t \phi^\star_x \nabla \phi_x + \partial_t \phi_x \nabla \phi^\star_x) \;, \quad \boldsymbol{E}(t, \boldsymbol{x}) = \delta \left\langle \widehat{\boldsymbol{E}}(t, \boldsymbol{x}) \right\rangle \;. \tag{20}$$

The response of such a current to the gradients is, to first order in the gradients, given by

$$\delta \left\langle \boldsymbol{j}(t, \boldsymbol{x}) \right\rangle = \mathrm{i} \int_{\mathrm{t_0}}^{\mathrm{t}} \mathrm{d}\tau \left\langle \left[\boldsymbol{j}(\mathrm{t} - \tau, \boldsymbol{x}), \widehat{\mathcal{Q}} \right] \right\rangle \;. \tag{21}$$

The commutator contains *four* field operators instead of *two*. The expectation value of this commutator therefore cannot be expressed completely through the commutator function of generalized free fields. Hence in the following, we neglect higher order correlations and use the canonical commutation relations of the ξ-operators to obtain in the stationary limit

$$\boldsymbol{j}_l(t,\boldsymbol{x}) = \\ -2\pi\mathrm{i}\int_0^\infty \mathrm{dE}\int \frac{\mathrm{d}^3\boldsymbol{k}\,\mathrm{d}^3\boldsymbol{k}'}{(2\pi)^3}\,\mathcal{A}_\mathrm{l}(\mathrm{E},\boldsymbol{k})\,\mathcal{A}_\mathrm{l}(\mathrm{E},\boldsymbol{k}')\,\mathrm{e}^{\mathrm{i}(\boldsymbol{k}-\boldsymbol{k}')\boldsymbol{x}}\,(\boldsymbol{k}+\boldsymbol{k}')\,\mathcal{P}_\mathrm{l}(\boldsymbol{k},\boldsymbol{k}') \quad (22)$$

for each species. The energy current is formally quite similar, and the total currents are for both cases

$$\begin{aligned}\boldsymbol{j}(t,\boldsymbol{x}) &= \boldsymbol{j}_+(t,\boldsymbol{x}) - \boldsymbol{j}_-(t,\boldsymbol{x}) \\ \boldsymbol{E}(t,\boldsymbol{x}) &= \boldsymbol{E}_+(t,\boldsymbol{x}) + \boldsymbol{E}_-(t,\boldsymbol{x})\,. \end{aligned} \quad (23)$$

In principle the above expressions can be calculated, when the spectral functions and the space-dependence of n_l are given. However, it is better justified to perform a gradient expansion for the occupation number density n_l, since the usual spectral function is correct also only to first order in these gradients. The i-th vector component of the l-charged currents generated by the inhomogeneity of the system is then

$$\begin{aligned}\boldsymbol{j}_l^{(i)}(t,\boldsymbol{x}) &= 2\pi\int\frac{d^3\boldsymbol{k}}{(2\pi)^3}\,\frac{\boldsymbol{k}^{(i)}}{2\,Z_{Ql}^2}\quad\int dE\;\left(\mathcal{A}_l(E,\boldsymbol{k})\right)^2\times \\ &\qquad\int dE'\,E'\left\{\frac{\partial n_l(E',\boldsymbol{k},\boldsymbol{x})}{\partial\boldsymbol{x}^{(j)}}\frac{\partial\mathcal{A}_l(E',\boldsymbol{k})}{\partial\boldsymbol{k}^{(j)}}\right\} \quad (24)\\ \boldsymbol{E}_l^{(i)}(t,\boldsymbol{x}) &= 2\pi\int\frac{d^3\boldsymbol{k}}{(2\pi)^3}\,\frac{\boldsymbol{k}^{(i)}}{2\,Z_{Ql}^2}\quad\int dE\,E\;\left(\mathcal{A}_l(E,\boldsymbol{k})\right)^2\times \\ &\qquad\int dE'\,E'\left\{\frac{\partial n_l(E',\boldsymbol{k},\boldsymbol{x})}{\partial\boldsymbol{x}^{(j)}}\frac{\partial\mathcal{A}_l(E',\boldsymbol{k})}{\partial\boldsymbol{k}^{(j)}}\right\}\,. \quad (25)\end{aligned}$$

Before this is evaluated further, note that the expression in the curly brackets is nothing but the first term of a full gradient expansion of the product of n_l and $\mathcal{A}_l$, i.e., the Poisson bracket. It is therefore clear, that the above expression is useful also for situations, where the space-dependence of $\mathcal{A}_l$ cannot be neglected: the Poisson bracket then also has a contribution with a space-derivative acting on $\mathcal{A}_l$ and a momentum derivative acting on n_l. The above expression is also interesting in view of the original equation (21), since the Poisson bracket is the analogon of the commutator, i.e., it contributes a factor $\hbar$ to the current.

Using as local equilibrium spectral function the one for global equilibrium (correct to first order in the gradients, cf. [8]), and as distribution function

$$n_\pm(E,\boldsymbol{k},\boldsymbol{x}) = \frac{f_\pm(E,\boldsymbol{x})}{1-f_\pm(E,\boldsymbol{x})} = \frac{1}{\exp\left(\beta(\boldsymbol{x})(E\mp\mu(\boldsymbol{x}))\right)-1}\,, \quad (26)$$

the derivative with respect to $\boldsymbol{x}$ has two components: Along the temperature gradient and along the gradient of μ.

The same decomposition into T and μ-gradient terms then holds for the currents, i.e., with transport coefficients L_{ij} we obtain

$$\begin{aligned} \boldsymbol{j}(t,\boldsymbol{x}) &= -L_{11}\nabla\mu \quad -L_{12}\frac{\nabla T}{T} \\ \boldsymbol{E}(t,\boldsymbol{x}) &= -L_{21}\nabla\mu \quad -L_{22}\frac{\nabla T}{T}\ . \end{aligned} \tag{27}$$

Note, that L_{12} and L_{21} can be different, i.e. the Onsager relation $L_{ij} = L_{ji}$ is not necessarily fulfilled. The reason for this is clear: in contrast to ordinary many-body quantum physics, the present formulation exhibits dissipation already on the tree-graph level [1]. In other words, the formulation of an interacting theory with continuous spectral functions is *not* micro-reversible, physical states have a finite lifetime. However, the *complete* system, including all species of particles and interactions, by definition has a single partition function. Hence, the *global* Onsager relations are fulfilled.

From the above currents, one obtains the thermal conductivity for the interacting scalar field by subtracting the convective part

$$\lambda = \frac{1}{T}\left(L_{22} - \frac{L_{12}L_{21}}{L_{11}}\right)\ . \tag{28}$$

$L_{11} = d$ is the diffusion coefficient (or mobility). Before considering numerical results obtained with these equations, let us study the case of a simple spectral function

$$\mathcal{A}_+(E,\boldsymbol{k}) = \mathcal{A}_-(E,\boldsymbol{k}) = \frac{2E\gamma_k}{\pi}\,\frac{1}{(E^2-\Omega_k^2)^2 + 4E^2\gamma_k^2} \tag{29}$$

with $\Omega_k^2 = \epsilon_k^2 + \gamma_k^2$ and $\gamma_k \ll \epsilon_k$. In the absence of a chemical potential, L_{12} and L_{21} are zero, but diffusion coefficient and thermal conductivity are, to lowest order in the width for $\mu = 0$,

$$\begin{aligned} d_0 &= -\frac{1}{T}\int\frac{d^3\boldsymbol{k}}{(2\pi)^3}\,\frac{1}{3\gamma_k}\,\frac{\partial}{\partial k}\,\epsilon_k\, n(\epsilon_k)\,(1+n(\epsilon_k)) \\ \lambda_0 &= -\frac{1}{T}\int\frac{d^3\boldsymbol{k}}{(2\pi)^3}\,\frac{k\,\epsilon_k}{3\gamma_k}\,\frac{\partial}{\partial k}\,\epsilon_k\, n(\epsilon_k)\,(1+n(\epsilon_k))\ . \end{aligned} \tag{30}$$

Here, $n(\epsilon_k)$ is the local Bose function taken at energy ϵ_k. Apart from the momentum factors, a similar representation for λ_0 was obtained in ref. [12]. The difference can be attributed to the fact that in this reference a hydrodynamical picture was assumed together with $\beta\gamma_k \ll 1$. We thus make a comparison in the high temperature limit, where one obtains

$$\lim_{T\to\infty}\lambda_0 = \frac{A}{\bar{\gamma}}\,T^3\ . \tag{31}$$

Here, $\bar{\gamma}$ is some momentum averaged width and A a numerical factor. The same limiting behavior is obtained in [12].

The above expressions are also in accordance with naive expectations: The thermal conductivity and diffusion coefficient diverge, if the width of the particles is reduced, i.e., if the interaction is removed. In this case the time needed for the relaxation of a temperature disturbance is infinite.

As stated above, The Poisson bracket contributes a factor $\hbar$, while in a naive counting the inverse width contributes a factor $1/\hbar$ to the transport coefficient. This cancellation seems to be the main source for the difficulties one has in obtaining kinetic coefficients from quantum field theory.

5. Pionic transport properties

In the next step, we apply the model to pions in a homogeneous gas of nucleons and Δ_{33}-resonances. Instead of the above simplistic form, the more realistic pionic spectral function from ref. [9] is used. It exhibits a strong mixing of the pion with a collective exciation of Δ-particle/nucleon-hole pairs, hence has a richer structure with two resonance branches. In the figure, the product λT and and the diffusion coefficient d are plotted for this spectral function.

The high temperature behavior at the example density of $\rho = 1.59\times$ nuclear matter density inferred from the figure is

$$\lambda \approx 9897\,\mathrm{GeV/fm^2} \times [T/\mathrm{GeV}]^{3.70} \ , \tag{32}$$

i.e., it rises even faster than estimated by eqn. (31). This is due to the decreasing width of the pion with temperature in the Δ-hole model – a fact, which was not included in eqn. (31). At lower temperatures, the thermal conductivity rises slower with temperature than the asymptotic expression.

The density dependence of the coefficients λ and d is quite small, with the general tendency to have a "stiffer" temperature dependence at lower baryon density. This is consistent with the fact, that the pion in the Δ-hole model becomes a free particle without the baryonic background.

A comparison of these values to *experimental* data is beyond reach for the time being, we thus have to restrict the comparison to other calculations: The thermal conductivity in this field-theoretical calculation is substantially higher, than obtained in a fluid-dynamical picture fitted to pion scattering data at zero temperature and density[13,14].

This is consistent with the finding, that *off-shell effects*, which were treated consistently in the present approach, may substantially increase relaxation times [15]. Hence, within the framework of quantum field theory, the new method presented here gives reliable results for transport coefficients.

Top panel: Thermal conductivity $\times$ temperature.
Bottom panel: Diffusion coefficient.
Thin straight lines: slopes $\propto T^3$, T^4
Full thick line: Δ-hole model at $\rho = 1.59$ nuclear density;
dashed line 0.92 and dash-dotted line 0.47 nuclear density.
Dash-double-dotted in top panel [14], dotted line ref. [13],
both from $\pi - \pi$ scattering data at $T = 0$, $\rho = 0$.

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